THE STUDY OF DYNAMIC THERMAL 
MATHEMATICAL MODEL FOR EHV XLPE CABLE

Aihong TANG\(^1\), Yangyang ZHAO\(^1\), Desheng JIANG\(^2\)

\(^1\)Automation School, Wuhan University of Technology, China
E-mail: tah@whut.edu.cn, weilaizhy@163.com

\(^2\)Fiber Optic Sensing Technology Research Center, WuHan University of Technology, China
E-mail: jiangdsh@cae.cn

Abstract—This paper analyzes the dynamic behavior of the cable, deduces the dynamic mathematical model of XLPE based on the equation of thermal circuit, for the operation supervising of the cable, programs a program, the program can calculate the rated current-carry capacity of the cable and the thermal dynamic process of the cable. The calculation results verify the dynamic mathematical model of the XLPE presented in this paper.

Key words: Terms-Thermal field, thermal circuit, thermal dynamic mathematical model, current-carry capability

I. INTRODUCTION

Because of their structure and materials are unique, when transmission steady-state or dynamic load, the maximum operating temperature of the conductor of the ultra-high voltage transmission cable line has strictly controlled. Too high Conductor operating temperature will change the structure of cables Ontology hot field distribution, accelerate the aging process of the thermal insulating materials, cause bending and fixed parts of thermoplastic deformation, and will result in distortion of electric field distribution or concentration, effect the safe operation of the power cable lines, further more, even a security power cable will run accident.

Therefore, for the further research of the EHV cable operation characteristics of the load, there is a need to study the dynamic mathematical model of the ultra-high voltage cable lines with heat variable loads, and provide the practical enriched theoretical basis and technical basis for the current-carry capability of the ultra-high voltage cable.

This paper will consider all of the factors which impact the thermal process to establish the thermal mathematical model of the EHV XLPE thermodynamic process.

II. THERMODYNAMIC BEHAVIOUR ANALYZE

During the operation of the power cable, because of the current carried by the cable, there will result in the power loss in the core conductor, insulation layer, sheath screen and the armour. All of these power losses will result in the thermal augment in the cable, and this will increase the temperature of the cable. These heats emitting out sources are called thermal source. There will form a thermal field because of the existence of the thermal source. The temperature of anything in the thermal field will increase. At any point of the thermal field, the thermal current \(dW\) running through the unite area \(dA\) will result in the temperature gradient \(\frac{d\theta}{dh}\), the relationship can be stated as [1]

\[
\frac{dW}{dA} = -\lambda\left(\frac{d\theta}{dh}\right)dA
\]

where, \(\lambda\) is the heat conductance coefficients. Because almost all of the cables are cylindrical, in the thermal field calculation of power cable, we can think that, the heat emitting only along the radial. The cylindrical radial structure is shown in Figure1. in the point of \(x\) among the insulation, take a unit long cylindrical with the...
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thick of \( dx \), the volume of the cylindrical is
\[ dV = 2\pi x \cdot dx \cdot 1. \]

If the thermal current runs into this area in unit time is \( W \), then, the current out of this area is
\[ W + (dW / dx) \cdot dx. \]

If \( q \) is the element thermal circuit coefficient, when the temperature is increased as \( \frac{d\theta}{dt} \), the absorbed thermal energy by this area in unit time is
\[ qdV (d\theta / dt). \]

Take the symbol \( W_i \) as the emitted out thermal energy in unit area within unit time, we can get the following equation with reference to conservation of energy theorem and the continuous heat theorem.

\[ W + W_i dV = (dW / dx)dx + W + qdV (d\theta / dt) \]  \hspace{1cm} (2)

Taking into accounting of (1), we can get
\[ \frac{dW}{dx} = -2\pi \lambda (x(d^2 \theta / dx^2)) + (1 / x) \]
\[ \times (d\theta / dx)) + W_i / q \]  \hspace{1cm} (3)

Put Eqn (3) into Eqn (2), we can get the following thermal field equation, which can describe the heat process of the power cable.

\[ d\theta / dt = \left( \lambda / q \right)^* \left( (d^2 \theta / dx^2) + (1 / x) \right) \]
\[ \times (d\theta / dx)) + W_i / q \]  \hspace{1cm} (4)

The thermal field Eqn (4) indicates that, in steady-state, the temperature of the cable will not vary with the load current; the emitting speed is increased with the bigger the element thermal circuit; if the heat emitting speed is decreased, the temperature rise in unit will be a little slow. In practice, in order to deduce the calculation equation for the current-carry capability, the thermal field is translated into the thermal circuit first.

III. MATHEMATICAL MODEL

Usually, from inner to outer, the high-voltage XLPE cable is constructed as conductor, semiconductor screen, insulate material, outer semiconductor screen, belt protection and armoured sheath [2]. So, the thermal dynamic mathematical model of the XLPE of the vertical cooling surface can be equalised as Figure 2.

![Fig. 1. The cylindrical radial structure](image)

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![Fig. 2. Dynamic mathematical model of XLPE](image)

In the Figure 2, \( T_1 \) is the thermal resistance per core between conductor and sheath, \( K \cdot m/W \); \( T_2 \) is the thermal resistance between sheath and armour, \( K \cdot m/W \); \( T_3 \) is the thermal resistance of external serving, \( K \cdot m/W \); \( T_4 \) is the thermal resistance of surrounding medium (ratio of cable surface temperature rise above ambient to the losses per unit length), \( K \cdot m/W \). \( \theta_c \) is the operating temperature of conductor; \( \theta_o \) is the temperature of soil between dry and moist zones, \( \theta_m \) is the operating temperature of cable screen or sheath, \( \theta_s \) is the operating temperature of armour, \( \theta_e \) is the ambient temperature, \( W(t) \) is losses in conductor per unit length, W/m. \( Q_1 \), \( Q_2 \), \( Q_3 \), \( Q_4 \) is the element of thermal circuit of cable, their values can be get through the following equations,

\[ \begin{align*}
Q_1 &= Q_0 + pQ \\
Q_2 &= (1 - p)Q + pQ_1 + Q_{0p} \\
Q_3 &= (1 - p)Q_1 \\
Q_4 &= (1 - p)Q
\end{align*} \]  \hspace{1cm} (5)

where, \( Q_0 \) is the conductor element thermal circuit of cable, \( Q_1 \) is the insulate element thermal circuit of cable, \( Q_2 \) is the screen and
sheath element thermal circuit of cable, \( Q_{cp} \) is the armour element thermal circuit of cable, \( Q_e \) is element thermal of environment.

\[
\begin{align*}
\varphi &= \frac{1}{2} \ln \left( \frac{D_i}{d_c} \right) - \frac{1}{2} \ln \left( \frac{D_i}{d_c} - 1 \right) \\
\varphi' &= \frac{1}{2} \ln \left( \frac{D_c}{D_i} \right) - \frac{1}{2} \ln \left( \frac{D_c}{D_i} - 1 \right) 
\end{align*}
\]  

(6)

where, \( D_i \) is the diameter over insulation, \( d_c \) is the diameter of the conductor, \( D_s \) is the external diameter of metal sheath, \( D_e \) is the external diameter of cable.

Based on the dynamic mathematical model of the figure2, we can calculate the transient temperature response. There are two steps in the calculation of transient temperature response: the first step is for the transient temperature rise caused by the outer surface of the cable, the second step is for the transient temperature rise caused by the surrounding medium of the cable. Cable transient temperature rising for a complete cable is composed by the temperature rise of the surrounding medium multiplied by the transient temperature rise factor and the temperature rise of ontology of the cable.

III. TEST RESULTS

For the operation supervising of the cable, we compose a program for the current-carrying capacity of a cable. According to the IEC-60287 [3] and IEC-60853[4], the calculation is divided into three steps, the first step is the power loss of all layers of the cable, the second one is the thermal resistance calculation, the third is the current-carry capability calculation. The inputs include the cable parameters, the bury environment and the bonded model of the sheath; the outputs are the constracter parameters of the cable, the power loss, the thermal parameters, and the load information.

The program is composed by the VB in Chinese. The starting interface is shown in Figure3. From this, we can get the curves about the maximum temperature, maximum load and maximum current-carry capacity of top 10. the interface for the thermal supervisor is shown in Figure4, it shows the maximum external temperature of the cable, the maximum short-circuit current can carry by the cable at current temperature, and so on.

In order to verify the dynamic thermal mathematical model established by this paper, we select the YJV as test example, the voltage of the cable is 8.7/15kVA, \( D_i \) is 35.6mm, \( d_c \) is 23.6mm, \( D_s \) is 35.8mm, \( D_e \) is 41mm. the sheaths are bonded in two ends, buried in soil.

When we carry the rated current, the maximum external temperature is about 70℃, and in this situation the maximum short-circuit current is about 44896.64A. This is almost the same to the example in [5].

Fig.3. starting interface of the program

Fig.4. thermal supervisor interface

V. CONCLUSIONS

This paper analysis the thermodynamic behaviour of the high voltage XLPE cable, deduces the dynamic mathematical model with the consideration of the element thermal circuit of the cable when the carried-current is varied. Tests based on current-carrying capacity calculation software verify the accuracy of the thermal mathematical model established by this paper. This will be very useful for the cable load monitoring

REFERENCES

BIOGRAPHY

Aihong TANG was born in JIANGXI province of China on December 1, 1969. She received her Ph.D for power systems and its automation from Huazhong University in June, 2007, and MS for power systems and its automation from Wuhan University of hydraulic and electric in June, 1997. Her employment experience included the power supply bureau, electric power design institute and university. Now, she engages in the operation and intelligent controlling for power systems and flexibility AC transmission systems.